Group Homework Solutions:

1.8

1.8 The volume rate of flow, Q, through a pipe containing a slowly moving liquid is given by the equation

$$Q = \frac{\pi R^4 \Delta p}{8 \mu \ell}$$

where R is the pipe radius, Δp the pressure drop along the pipe, μ a fluid property called viscosity $(FL^{-2}T)$, and ℓ the length of pipe. What are the dimensions of the constant $\pi/8$? Would you classify this equation as a general homogeneous equation? Explain.

$$[L^3 T^{-1}] \doteq \left[\frac{\pi}{8}\right] \frac{[L^4][FL^{-2}]}{[FL^{-2}T][L]}$$

$$[L^3 T^{-1}] \doteq \left[\frac{\pi}{8}\right] [L^3 T^{-1}]$$

The constant $\pi/8$ is dimensionless, and the equation is a general homogeneous equation that is valid in any consistent unit system. Yes.

1.21

1.21 A tank of oil has a mass of 25 slugs. **(a)** Determine its weight in pounds and in newtons at the earth's surface. **(b)** What would be its mass (in slugs) and its weight (in pounds) if located on the moon's surface where the gravitational attraction is approximately one-sixth that at the earth's surface?

(a) weight = mass × g

$$= (25 \text{ slugs})\left(32.2 \frac{ft}{s^2}\right) = \underline{\underline{805 \text{ lb}}}$$

$$= (25 \text{ slugs})\left(14.59 \frac{kg}{slug}\right)\left(9.81 \frac{m}{s^2}\right) = \underline{\underline{3580 \text{ N}}}$$

(b) mass = 25 slugs (mass does not depend on gravitational attraction)

$$\text{weight} = (25 \text{ slugs})\left(\frac{32.2 \frac{ft}{s^2}}{6}\right) = \underline{\underline{134 \text{ lb}}}$$

1.29

1.29 The information on a can of pop indicates that the can contains 355 mL. The mass of a full can of pop is 0.369 kg while an empty can weighs 0.153 N. Determine the specific weight, density, and specific gravity of the pop and compare your results with the corresponding values for water at 20 °C. Express your results in SI units.

$$\gamma = \frac{\text{weight of fluid}}{\text{volume of fluid}} \qquad (1)$$

total weight = mass × g = $(0.369\ kg)(9.81\ \frac{m}{s^2}) = 3.62\ N$

weight of can = $0.153\ N$

volume of fluid = $(355 \times 10^{-3}\ L)(10^{-3}\ \frac{m^3}{L}) = 355 \times 10^{-6}\ m^3$

Thus, from Eq. (1)

$$\gamma = \frac{3.62\ N - 0.153\ N}{355 \times 10^{-6}\ m^3} = \underline{\underline{9770\ \frac{N}{m^3}}}$$

$$\rho = \frac{\gamma}{g} = \frac{9770\ \frac{N}{m^3}}{9.81\ \frac{m}{s^2}} = 996\ \frac{N \cdot s^2}{m^4} = \underline{\underline{996\ \frac{kg}{m^3}}}$$

$$SG = \frac{\rho}{\rho_{H_2O@4°C}} = \frac{996\ \frac{kg}{m^3}}{1000\ \frac{kg}{m^3}} = \underline{\underline{0.996}}$$

For water at 20°C (see Table B.2 in Appendix B)

$$\gamma_{H_2O} = 9789\ \frac{N}{m^3}\ ;\ \rho_{H_2O} = 998.2\ \frac{kg}{m^3}\ ;\ SG = 0.9982$$

A comparison of these values for water with those for the pop shows that the specific weight, density, and specific gravity of the pop are all slightly lower than the corresponding values for water.

1.36

1.36 A tire having a volume of 2.5 ft³ contains air at a gage pressure of 30 psi and a temperature of 70 °F. Determine the density of the air and the weight of the air contained in the tire.

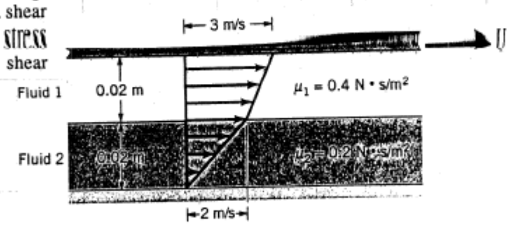
$$\rho = \frac{p}{RT} = \frac{(30\ \frac{lb}{in.^2} + 14.7\ \frac{lb}{in.^2})(144\ \frac{in.^2}{ft^2})}{(1716\ \frac{ft \cdot lb}{slug \cdot °R})[(70°F + 460)°R]} = \underline{\underline{7.08 \times 10^{-3}\ \frac{slugs}{ft^3}}}$$

$$\text{weight} = \rho g \times \text{volume} = (7.08 \times 10^{-3}\ \frac{slugs}{ft^3})(32.2\ \frac{ft}{s^2})(2.5\ ft^3)$$

$$= \underline{\underline{0.570\ lb}}$$

1.54

1.54 As shown in **Video V1.2**, the "no slip" condition means that a fluid "sticks" to a solid surface. This is true for both fixed and moving surfaces. Let two layers of fluid be dragged along by the motion of an upper plate as shown in Fig. P1.54. The bottom plate is stationary. The top fluid puts a shear stress on the upper plate, and the lower fluid puts a shear stress on the botton plate. Determine the ratio of these two shear stresses.



■ FIGURE P1.54

For fluid 1

$$\tau_1 = \mu_1 \left(\frac{du}{dy}\right)_{top\ surface} = \left(0.4\ \frac{N \cdot s}{m^2}\right)\left(\frac{3\frac{m}{s} - 2\frac{m}{s}}{0.02 m}\right) = 20\ \frac{N}{m^2}$$

For fluid 2

$$\tau_2 = \mu_2 \left(\frac{du}{dy}\right)_{bottom\ surface} = \left(0.2\ \frac{N \cdot s}{m^2}\right)\left(\frac{2\frac{m}{s}}{0.02\ m}\right) = 20\ \frac{N}{m^2}$$

Thus,

$$\frac{\tau_{top\ surface}}{\tau_{bottom\ surface}} = \frac{20\ \frac{N}{m^2}}{20\ \frac{N}{m^2}} = \underline{\underline{1}}$$

1.56

1.56 The sled shown in Fig. P1.56 slides along on a thin horizontal layer of water between the ice and the runners. The horizontal force that the water puts on the runners is equal to 1.2 lb when the sled's speed is 50 ft/s. The total area of both runners in contact with the water is 0.08 ft², and the viscosity of the water is 3.5×10^{-5} lb s/ft². Determine the thickness of the water layer under the runners. Assume a linear velocity distribution in the water layer.

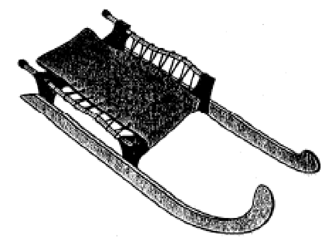


FIGURE P1.56

F (force) $= \tau A$

$\tau = \mu \frac{dv}{dy} = \mu \frac{V}{d}$ where d = thickness of water layer

Thus,

$$F = \mu \frac{V}{d} A$$

and

$$d = \frac{\mu V A}{F} = \frac{\left(3.5 \times 10^{-5} \frac{lb \cdot s}{ft^2}\right)\left(50 \frac{ft}{s}\right)\left(0.08\, ft^2\right)}{1.2\ lb}$$

$$= \underline{\underline{11.7 \times 10^{-4}\ ft}}$$

1.59

1.59 A layer of water flows down an inclined fixed surface with the velocity profile shown in Fig. P1.59. Determine the magnitude and direction of the shearing stress that the water exerts on the fixed surface for $U = 2$ m/s and $h = 0.1$ m.

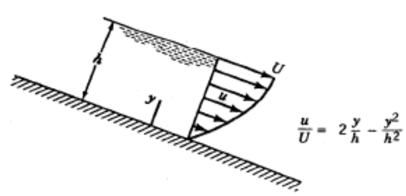


FIGURE P1.59

$$\tau = \mu \frac{du}{dy}$$

$$\frac{du}{dy} = U\left(\frac{2}{h} - \frac{2y}{h^2}\right)$$

Thus, at the fixed surface ($y=0$)

$$\left(\frac{du}{dy}\right)_{y=0} = \frac{2U}{h}$$

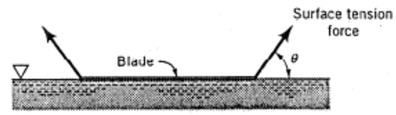
so that

$$\tau = \mu\left(\frac{2U}{h}\right) = \left(1.12 \times 10^{-3} \frac{N \cdot s}{m^2}\right)(2)\frac{\left(2 \frac{m}{s}\right)}{(0.1 m)}$$

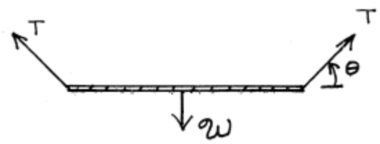
$$= \underline{\underline{4.48 \times 10^{-2} \frac{N}{m^2} \text{ acting in direction of flow}}}$$

1.84

1.84 As shown in Video V1.5, surface tension forces can be strong enough to allow a double-edge steel razor blade to "float" on water, but a single-edge blade will sink. Assume that the surface tension forces act at an angle θ relative to the water surface as shown in Fig. P1.84. **(a)** The mass of the double-edge blade is 0.64×10^{-3} kg, and the total length of its sides is 206 mm. Determine the value of θ required to maintain equilibrium between the blade weight and the resultant surface tension force. **(b)** The mass of the single-edge blade is 2.61×10^{-3} kg, and the total length of its sides is 154 mm. Explain why this blade sinks. Support your answer with the necessary calculations.



■ FIGURE P1.84



(a) $\Sigma F_{vertical} = 0$

$$\mathcal{W} = T \sin\theta$$

where $\mathcal{W} = m_{blade} \times g$ and $T = \sigma \times$ length of sides.

$$\therefore \; (0.64 \times 10^{-3} \, kg)(9.81 \, m/s^2) = (7.34 \times 10^{-2} \, \tfrac{N}{m})(0.206 \, m) \sin\theta$$

$$\sin\theta = 0.415$$

$$\underline{\underline{\theta = 24.5°}}$$

(b) For single-edge blade

$$\mathcal{W} = m_{blade} \times g = (2.61 \times 10^{-3} \, kg)(9.81 \, m/s^2)$$
$$= 0.0256 \, N$$

and

$$T \sin\theta = (\sigma \times \text{length of blade}) \sin\theta$$
$$= (7.34 \times 10^{-2} \, N/m)(0.154 \, m) \sin\theta$$
$$= 0.0113 \sin\theta$$

In order for blade to "float" $\mathcal{W} < T\sin\theta$. Since maximum value for $\sin\theta$ is 1, it follows that $\mathcal{W} > T\sin\theta$ and single-edge blade will sink.

Solutions for Extra Practice:

1.12

$$Q = C\sqrt{2g}\,B\left(H + V^2/2g\right)^{3/2}$$

$$\left[L^3T^{-1}\right] \doteq \left[C\right]\left[L^{1/2}T^{-1}\right]\left[L\right]\left[L^{3/2}\right]$$

Since each term must have the same dimensions, C is <u>dimensionless</u>. Thus the equation is a general homogeneous equation that would be valid in any consistent system of units. <u>Yes.</u>

1.28

1.28 A beaker contains 10 in.^{3} of pure glycerin. If 2 in.^{3} of water is added to the glycerine, what is the specific gravity of the mixture?

$$\text{density of mixture} = \frac{\rho_{gly} \times (volume)_{gly} + \rho_{H_2O} \times (volume)_{H_2O}}{(volume)_{gly} + (volume)_{H_2O}}$$

$$= \frac{\left[\left(2.44\ \frac{slugs}{ft^3}\right)(10\ in.^3) + \left(1.94\ \frac{slugs}{ft^3}\right)(2\ in.^3)\right]\left(\frac{1\ ft^3}{1728\ in.^3}\right)}{(10\ in.^3 + 2\ in.^3)\left(\frac{1\ ft^3}{1728\ in.^3}\right)}$$

$$= 2.36\ \frac{slugs}{ft^3}$$

$$SG = \frac{\rho}{\rho_{H_2O\,@\,4^\circ C}} = \frac{2.36\ \frac{slugs}{ft^3}}{1.94\ \frac{slugs}{ft^3}} = \underline{\underline{1.22}}$$

1.34

1.34 A closed tank having a volume of 2 ft^3 is filled with 0.30 lb of a gas. A pressure gage attached to the tank reads 12 psi when the gas temperature is 80 °F. There is some question as to whether the gas in the tank is oxygen or helium. Which do you think it is? Explain how you arrived at your answer.

Density of gas in tank $\rho = \dfrac{\text{weight}}{g \times \text{volume}} = \dfrac{0.30\ lb}{\left(32.2\ \frac{ft}{s^2}\right)(2\ ft^3)}$

$$= 4.66 \times 10^{-3}\ \frac{slugs}{ft^3}$$

Since $\rho = \dfrac{p}{RT}$ with $p = (12 + 14.7)$ psia

(atmospheric pressure assumed to be $\approx$ 14.7 psia)

and with $T = (80°F + 460)\,°R$ it follows that

$$\rho = \frac{\left(26.7\ \frac{lb}{in.^2}\right)\left(144\ \frac{in.^2}{ft^2}\right)}{R\,(540°R)} = \frac{7.12}{R}\ \frac{slugs}{ft^3} \qquad (1)$$

From Table 1.7 $R = 1.554 \times 10^3$ for oxygen

and $R = 1.242 \times 10^4\ \frac{ft \cdot lb}{slug \cdot °R}$ for helium.

Thus, from Eq. (1) if the gas is oxygen

$$\rho = \frac{7.12}{1.554 \times 10^3}\ \frac{slugs}{ft^3} = 4.58 \times 10^{-3}\ \frac{slugs}{ft^3}$$

and for helium

$$\rho = \frac{7.12}{1.242 \times 10^4} = 5.73 \times 10^{-4}\ \frac{slugs}{ft^3}$$

A comparison of these values with the actual density of the gas in the tank indicates that the gas must be <u>oxygen</u>.

1.57

1.57 A 25-mm-diameter shaft is pulled through a cylindrical bearing as shown in Fig. P1.57. The lubricant that fills the 0.3-mm gap between the shaft and bearing is an oil having a kinematic viscosity of $8.0 \times 10^{-4}\ m^2/s$ and a specific gravity of 0.91. Determine the force P required to pull the shaft at a velocity of 3 m/s. Assume the velocity distribution in the gap is linear.

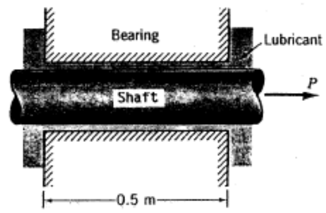
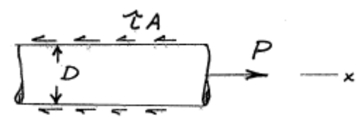


FIGURE P1.57



$\sum F_x = 0$

Thus,
$$P = \tau A$$

where $A = \pi D \times (\text{shaft length in bearing}) = \pi D \ell$

and
$$\tau = \mu \frac{(\text{velocity of shaft})}{(\text{gap width})} = \mu \frac{V}{b}$$

so that
$$P = \left(\mu \frac{V}{b}\right)(\pi D \ell)$$

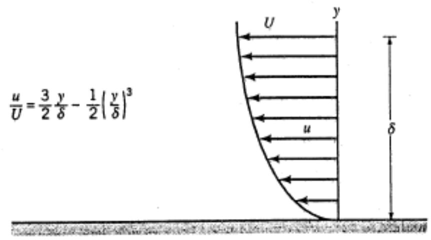
Since $\mu = \nu \rho = \nu (SG)(\rho_{H_2O @ 4°C})$,

$$P = \frac{(8.0 \times 10^{-4} \frac{m^2}{s})(0.91 \times 10^3 \frac{kg}{m^3})(3 \frac{m}{s})(\pi)(0.025m)(0.5m)}{(0.0003m)}$$

$$= \underline{\underline{286\ N}}$$

1.58

1.58 A Newtonian fluid having a specific gravity of 0.92 and a kinematic viscosity of $4 \times 10^{-4}\ \mathrm{m^2/s}$ flows past a fixed surface. Due to the no-slip condition, the velocity at the fixed surface is zero (as shown in Video V1.2), and the velocity profile near the surface is shown in Fig. P1.58. Determine the magnitude and direction of the shearing stress developed on the plate. Express your answer in terms of U and δ, with U and δ expressed in units of meters per second and meters, respectively.



■ FIGURE P1.58

$$\tau_{\substack{surface\\(y=0)}} = \mu \left(\frac{du}{dy}\right)_{y=0}$$

$$\frac{du}{dy} = U\left(\frac{3}{2\delta} - \frac{3}{2}\frac{y^2}{\delta^3}\right)$$

$$@\ y=0, \quad \frac{du}{dy} = \frac{3}{2}\frac{U}{\delta}$$

$$\text{Since, } \mu = \nu\rho$$

$$\tau_{surface} = \nu\rho\left(\frac{3}{2}\frac{U}{\delta}\right)$$

$$= \left(4\times10^{-4}\frac{m^2}{s}\right)\left(0.92\times10^3\frac{kg}{m^3}\right)\left(\frac{3}{2}\right)\frac{U}{\delta}$$

$$= \underline{\underline{0.552\frac{U}{\delta}\ N/m^2 \text{ acting to left on plate}}}$$

1.83

1.83 A 12-mm diameter jet of water discharges vertically into the atmosphere. Due to surface tension the pressure inside the jet will be slightly higher than the surrounding atmospheric pressure. Determine this difference in pressure.

For equilibrium (see figure),

$$p(2R\delta l) = \sigma(2\delta l)$$

so That

$$p = \frac{\sigma}{R}$$

$$= \frac{7.34\times10^{-2}\ \frac{N}{m}}{\frac{12}{2}\times10^{-3}\ m}$$

$$= \underline{\underline{12.2\ Pa}}$$